

SOUND AND ULTRASONICS

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SOUND

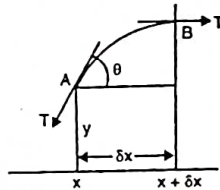
Transverse vibration of a stretched string

Expression for the velocity of transverse waves (First method)

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Consider a string lying along the X axis. m is the mass per unit length of the string. The string is kept under a tension T .

The string is slightly pulled aside along the Y axis and released. A transverse wave travels along the X axis. The displacements of the particles of the string take place along the Y axis.



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Consider a small segment AB of the displaced string (Figure). Its length is δx . The tension T in the string has the same magnitude everywhere. But the direction of T changes from point to point. The tensions at A and B will be tangential to the string at A and B. Let the tangent to the string at A make an angle θ with the X-axis. Component of the tension at A in the Y-direction is

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$$F_1 = T \sin \theta = T \tan \theta \quad (\because \sin \theta = \tan \theta \text{ when } \theta \text{ is small})$$

$$= T \frac{dy}{dx}$$

Component of the tension at B in the Y direction is

$$F_2 = T \frac{dy}{dx} + \frac{d}{dx} T \frac{dy}{dx} \delta x = T \frac{dy}{dx} + T \frac{d^2y}{dx^2} \delta x$$

But F_1 and F_2 are in opposite directions.

The resultant force on the segment AB = $F = F_2 - F_1$

$$= T \frac{d^2y}{dx^2} \delta x$$

The mass of the segment AB is $m \delta x$ and its acceleration is $\frac{d^2y}{dt^2}$

Force on the segment AB = $m \delta x \times \frac{d^2y}{dt^2}$

$$m \delta x \frac{d^2y}{dt^2} = T \frac{d^2y}{dx^2} \delta x$$

$$\frac{d^2y}{dt^2} = \frac{T}{m} \frac{d^2y}{dx^2}$$

This is the differential equation of a wave motion.

The velocity is given by $v = \sqrt{T/m}$

$$m = 0.090/3 = 0.030 \text{ kgm}^{-1}$$

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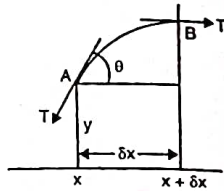
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$$\frac{d^2y}{dt^2} = \frac{T d^2y}{m dx^2}$$

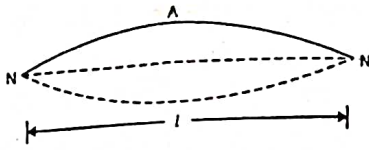
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Frequency of transverse vibration of a stretched string

Consider a string of length l , stretched under a tension T , fixed at its ends (figure).



Let m be the mass per unit length of the string. We pluck the string at its mid-point. The string vibrates in a single loop emitting its fundamental note. There are nodes at the fixed ends and an antinode at the centre.

$$l = \lambda/2$$

Wavelength of the stationary wave

$$\lambda = 2l$$

Now

$$v = n\lambda \text{ and } v = \sqrt{T/m}$$

$$n = v/\lambda$$

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

Laws of Transverse Vibration of Strings

- The fundamental frequency of a string is inversely proportional to the length of the string, when the tension and linear density are constant. $n \propto 1/l$ if T and m are kept constant or $nl = \text{constant}$.

$$n \propto \frac{1}{l}$$

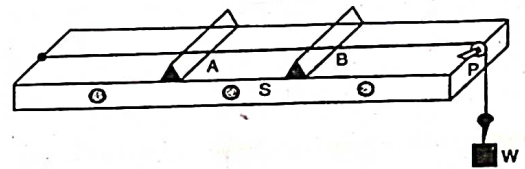
- The fundamental frequency of a string is directly proportional to the square root of the tension, when the length and linear density are constant. $n \propto \sqrt{T}$ if l and m are kept constant or $n/\sqrt{T} = \text{constant}$.

- The fundamental frequency of a string is inversely proportional to the square root of its linear density, when the length and tension are constant. $n \propto 1/\sqrt{m}$ if l and T are constant or $n\sqrt{m} = \text{constant}$.

Verification of the Laws of Transverse Vibration of Strings Sonometer

Description

The apparatus is shown in figure.



It consists of a thin metallic wire stretched across two movable bridges A and B on the top of a hollow, wooden sounding box (S') about one metre long. One end of the wire is fixed to a peg at one end of the box. The other end of the wire passes over a smooth fixed pulley (P) and carries a weight hanger W. The length of the vibrating segment of the wire can be altered with the help of the

movable bridges. The length of the vibrating segment can be measured by a scale fixed below the wire.

I Law :

A suitable tension T is applied to the sonometer wire. A small paper rider is placed on the wire. A tuning fork of known frequency is excited and its stem is pressed on the sounding box. The length of the wire is adjusted until the paper rider placed at the centre of the vibrating segment is thrown off. The length of the vibrating segment (l) is measured. Now, the frequency of the wire is the same as the frequency of the tuning fork. The experiment is repeated with forks of different frequencies, keeping the tension the same. The results are tabulated as shown.

No.	Frequency of the fork (a)	Length of vibrating segment (l)	nl

The product nl will be constant, thus verifying the first law.

II Law :

$n \propto \sqrt{T}$ when l and m are kept constant. It is difficult to verify the second law directly. Choose a tuning fork of frequency n . The wire is stretched by a load of 2 kg. The length of the wire vibrating in unison with the fork is determined. The experiment is repeated for loads of 3, 4 and 5 kg for the same fork. The readings are tabulated as shown below :

Frequency of the fork = _____ (Constant)

No.	Tension (T)	Length of vibrating segment (l)	\sqrt{T}/l

It is found that \sqrt{T}/l is a constant thereby verifying the law, indirectly As $n \propto \sqrt{T}/l$ and as n is kept the same, \sqrt{T}/l is a constant.

III Law

$n \propto l/\sqrt{m}$ when l and T are constant. It is difficult to verify the third law directly. For a convenient tension (say 3 kg), the length of the wire that vibrates in unison with a given tuning fork is determined. The experiment is repeated for three wires of different materials for the same tension and for the same fork. The linear density (m) of each wire is determined by finding the mass of about 1 metre of the wire. The results are tabulated as shown below

Tension = _____ (constant);

Frequency of the fork = _____ (constant)

No.	Linear density of the wire (m)	Length of vibrating segment (l)	l/\sqrt{m}

It is found that $l\sqrt{m}$ is a constant, thereby verifying the third law indirectly. As $n \propto 1/l\sqrt{m}$ and as n is kept the same, $l\sqrt{m}$ is constant.

Example : 1

Calculate the frequency of the fundamental note of a string 1 m long and weighing 2 grams when stretched by a weight of 400 kg.

Solution

$$l = 1 \text{ metre ;}$$

$$m = \frac{2 \times 10^{-3}}{1} = 2 \times 10^{-3} \text{ kg m}^{-1}$$

$$T = 400 \times 9.8 \text{ N} = 3920 \text{ N ; } n = ?$$

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}} = \frac{1}{2 \times 1} \sqrt{\frac{3920}{2 \times 10^{-3}}} = 700 \text{ Hz}$$

Example : 2

A wire gives out a fundamental note of 256 Hz when it is under a tension of 10 kg wt.

- (i) Under what tension will the string emit a frequency of 512 Hz?
- (ii) How would you make the wire emit a note of 768 Hz keeping the tension at 10 kg wt?

Solution

$$(i) n_1 = 256 \text{ Hz ; } T_1 = 10 \text{ kg wt ; } n_2 = 512 \text{ Hz ; } T_2 = ?$$

$$\frac{n_2}{n_1} = \sqrt{\frac{T_2}{T_1}}$$

$$\frac{512}{256} = \sqrt{\frac{T_2}{10}}$$

$$T_2 = 40 \text{ kg wt.}$$

$$(ii) n_1 = 256 \text{ Hz ; Length of the wire} = l_1$$

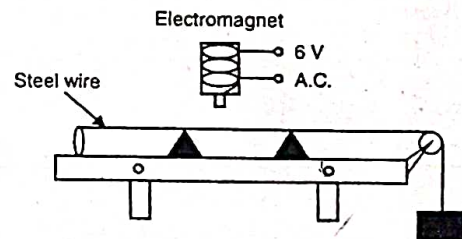
$$n_2 = 768 \text{ Hz ; Length of the wire} = l_2$$

$$\frac{n_2}{n_1} = \frac{l_2}{l_1} \text{ or } \frac{768}{256} = \frac{l_1}{l_2}$$

$$\text{or } l_2 = \frac{l_1}{3}$$

A.C. Frequency Measurement using Sonometer

The frequency of the alternating current mains in the laboratory can be determined using a sonometer.



Description

A sonometer consists of a thin uniform wire stretched over two bridges on a wooden box (figure). One end of the wire is fixed to a peg. The other end of the wire passes over a pulley and carries a weight hanger. The length of the vibrating segment of the wire can be altered with the help of the movable bridges. The length of the vibrating segment can be measured by a scale fixed below the wire.

Experiment

A steel wire is mounted on a sonometer under suitable tension. An electromagnet is excited by the low voltage alternating current whose frequency is to be determined. The electromagnet is placed just above the sonometer wire. The wire is attracted twice in each cycle.

A small paper rider is placed on the wire. The length of the wire is adjusted until the paper rider placed at the centre of the vibrating segment is thrown off. The length of the vibrating segment (l) is measured. The experiment is repeated for different tensions. The readings are tabulated as shown below :

No.	Tension (T)	Length of vibrating segment (l)	$\sqrt{T/l}$

The mean value of $\sqrt{T/l}$ is found.

The mass per unit length of wire is determined by finding the mass of a given length of the wire.

Calculation

The frequency of sonometer wire is

$$n = \frac{1}{2l} \sqrt{\frac{T}{m}}$$

During both the positive peak and the negative peak of the AC the wire is pulled by the electromagnet. So the wire vibrates twice, for each cycle of the AC.

The frequency of the AC supply is given by $f = n/2$

Hence, the frequency of AC mains is calculated.

ULTRASONIC WAVES

Introduction

When body vibrates, sounds are produced. If the frequency of vibration lies between 20Hz to 20,000Hz. the sound is audible to our ear. This range is known as audible range. If the frequency of sound wave is greater than 20,000 Hz it is not audible to our ear. These are known as ultrasonic or supersonic waves. The wavelength of ultrasonic waves are very small compared to audible sound. If

the frequency of the audible wave is 20 Hz then wavelength of the audible wave $\lambda = 330/20,000 = 0.165\text{m}$.

If the frequency of the ultrasonic wave is 20,000 Hz, the wavelength of the ultrasonic wave $\lambda = 330/20,000 = 0.165\text{m}$.

Ultrasonic waves can be detected by some birds like bats. The wavelength of the ultrasonic waves are very small. Hence it is applied in many field. The sound waves whose frequency is less than audible limit i.e. less than 20Hz are called infrasonics.

Production of ultrasonic waves:

Ultrasonic waves cannot be produced in ordinary way as sound waves are produced. In ordinary method, alternating current is given to the diaphragm of a loudspeaker and sound waves are produced. But if a high frequency current is given to a speaker, the inductive effect is large in the coil. Hence current will not flow through it. Moreover, the diaphragm of a loudspeaker cannot vibrate at such a high frequencies. Hence other methods are used produce ultrasonic waves.

Ultrasonic waves can be produced in the following ways.

- i. Galton whistle ✓
- ii. Magnetostriction oscillator ✓
- iii. Piezo - Electric oscillator ✓

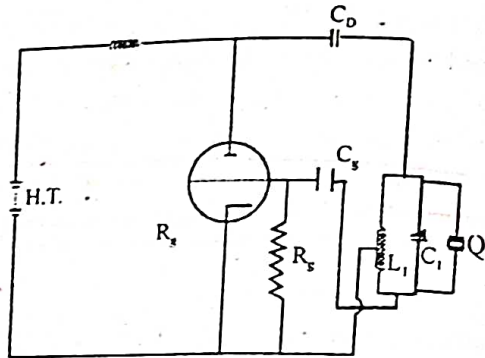
Piezo - Electric Oscillator:

This method is based on the phenomenon of piezo- electric effect. When certain crystals like quartz are stretched or compressed along certain axis, equal and opposite charges are produced on the faces of the perpendicular axis. Hence an electric potential difference is produced. This is known as piezo - electric effect.

The converse of this effect is also true. This principle is used in the piezo - electric oscillator. When an alternating potential is applied on the opposite faces of a crystal, the opposite faces in the perpendicular axis contracts and then elongates. When the frequency of the applied voltage is equal to the natural frequency of the crystal, the crystal vibrates with maximum amplitude. This property is used in the production of ultrasonic waves. The alternating potential difference is obtained by a valve oscillator.

Construction and working:

The experimental arrangement is as shown in figure. The high frequency alternating voltage which is applied to the crystal is obtained from Hartley oscillator. In Hartley oscillator, the tuned circuit consists of an inductance coil L_1 and a condenser C_1 in parallel. One end of the tuned circuit is connected to the anode and the other end to the grid. The cathode of the valve is connected to the center of the L_1 . The quartz crystal Q is connected in parallel with the variable capacitor C_1 .



Proper bias is given to the grid using the grid leak resistance R_g and the capacitor C_g . Using a radio frequency choke, d.c. voltage is given to the anode. The r.f.c. prevents the radio frequency current to pass the tank circuit. It by passes only the radio frequency currents. By adjusting the capacitance of the variable capacitor C_1 , the frequency of the oscillator is tuned to the natural frequency of the crystal. Now the quartz, crystal is set into mechanical vibration and ultrasonic waves are produced. By this method, ultrasonic waves of frequency 500KHz can be produced. Frequency upto 1.3×10^7 Hz can be produced using a tourmaline crystal.

Advantage

- i. Frequency as high as 500 MHz can be produced.
- ii. Stable and constant frequency can be generated.
- iii. Using synthetic material wide range of frequency can be produced at a lower cost.

- iv. The output is very high.
- v. It is sensitive to temperature and humidity.

Disadvantages

- i. Cost of quartz plate is high.
- ii. Cutting and shaping of crystal are very complex.

Properties of Ultrasonic wave

1. The frequency of the ultrasonic waves a greater than 20 KHz.
2. They are highly energetic.
3. The speed of ultrasonic waves depends upon their frequency. If the frequency is high, the speed also is high.
4. Since the wavelength is very small, the diffractions effect is negligible.
5. Intense ultrasonic waves have a disruptive effect on liquids by causing bubbles to be formed.
6. Ultrasonic waves are also reflected, refracted and absorbed just like ordinary sound wave.
7. When an ultrasonic wave are passed through a liquid, stationary wave patterns are formed due to the reflection of the wave from the other end. Along the direction of propagation, the density of the liquid varies from layer to layer. In this way, a plane diffraction grating is formed which can diffract light.

8. Ultrasonic waves are easily absorbed and hence it cannot be propagated to a long distance.

Applications of ultrasonics

- i. **Detection of aircraft, submarines etc.:** Since the frequency of ultrasonics waves are high it can be used for the detection of aircrafts, submarines, icebergs, or even other comparatively smaller objects on seabeds. High frequency waves are produced and it is sent through water. These waves are reflected by the solid objects. The reflected waves can be detected using a quartz receiver. From this, we can identify the objects. These are called sonar. (Sound Navigation and Ranging)
- ii. **Depth of the sea :** Echometer is a slight modification of Sonar. High frequency waves are produced and directed towards the bottom of the sea. The rays reflected by the bottom of the sea are received by the receiver. The time interval between the emitted signal and the echo can be determined. If the velocity of the sound wave is known, the depth of the sea can be calculated using the relation $h = Vt/2$.
- iii. **Signalling :** Ultrasonic waves are used for direction signaling. The narrow directional sound beam provided by the supersonic waves of a relatively high frequency is used for the purpose of signaling to a distant ship.
- iv. **Cracks in metals:** Ultrasonic waves be used to detect cracks or discontinuity in metal structures. When an ultrasonic wave

passed through a metal having cracks or discontinuity, they are reflected. Rays are also reflected by the other face of the metal. The reflected pulses received by a detector. From this, cracks, can be detected.

v. **Uses in industry:**

- a. Ultrasonic waves are used to bore holes in glass, steel and other alloys.
- b. These are used to remove grease, dust and metal filings from the manufacture parts of car, camera etc.
- c. These are used in cleaning clothes.
- d. Ultrasonic waves are used for soldreing the metals.

vi. **Chemical application:**

- a. Ultrasonic waves act like catalytic agents and accelerate chemical reactions.
- b. It is used in the preparation of photographic film and face cream etc.
- c. It is also used to produce a small uniform crystals.

vii. **Metallurgical use:** Many metals as Iron, lead aluminium, zinc etc which cannot be alloyed in their liquid state can be brought together when subjected to high intensity ultrasonic waves.

viii. **Acoustic microscope:** Using high frequency ultrasonic waves we can study the surface structure and its homogeneity with high resolution.

- ix. Acoustic holography: Acoustic holography is mainly used to study the surface structure of various engineering components used for space applications. This gives clear information about the micro cracks present on the surface of the specimen.
- x. In Medicine: Ultrasonic is one of the tools in medical diagnoses. It is used for the study of a variety of organs in areas like obstetrics and gynaecology, gastroenterology, cardiology, neurology, oncology, study of reticulo-endothelial system etc.

Ultrasonic finds application in the following also.

- a. Ultrasonic waves are used as a pain reliever. The waves produce a soothing massage action and relieves pain.
- b. Ultrasonic waves are used to restore the contracted fingers.
- c. These are used by dentists for proper extraction of broken teeth.
- d. Ultrasonic are use in bloodless surgery. The ultrasonic waves are focused on a sharp instrument and the tissues are destroyed without any loss of blood. These are also used for brain operations.

Acoustics of Buildings

Acoustics Buildings

Introduction

Buildings are constructed for some specific purpose. They should form according to their purposes. The branch of the science which deals with the design and construction of building with good acoustics is known acoustics of buildings or architectural acoustics.

W.C Sabine in 1911, studied in detail about the acoustics of a building. Now a half of auditorium are constructed with care about the acoustics. The following are the essential features about the good acoustics.

1. Every syllable or musical note should reach an adequate level of intensity at every point and then die away sufficiently quickly.
2. The quality of sounds must remain unaltered.
3. The successive sounds of speech must remain distinct and must be free from one another and from extraneous noise.
4. Echoes, except those required to maintain the necessary continuity, must be avoided.
5. Undesirable concentration of sound at one place and regions of poor audibility must be eliminated.
6. All the extraneous noises must be shut out as far as possible.
7. None of the subsequent reflections should complete in strength with direct sound.
8. The appropriate degree of reverberation should be provided and in particular the high frequencies must be preserved.
9. There should be no Echelon effect.
10. There should not be resonance within the building.

Factors Affecting the Acoustics of Building

The most important factor that affects the acoustics of buildings is the reverberation time. If it is large, there is overlapping of successive sounds which results in loss of clarity. If it is very small, the loudness is inadequate. Moreover, other factors that affect are loudness, focusing, extraneous noise, resonance etc.

With great absorption, the time of reverberation will be smaller. But the intensity of sound is weakened and may go below the level of hearing.

In an auditorium, electrically amplified loud speakers are used. Due to this, the intensity of sound is focused at a particular direction. If there are focusing surfaces like concave, spherical etc. on the walls, they produce concentration of sound at a particular direction. Hence there will be no uniform distribution of intensity of sound throughout the auditorium.

The sound waves are reflected in the steps of the stair. Each step reflects the sound wave in different time. It will produce undesirable echoes. This is called Echelon effect.

In a good hall, no noise should reach from outside. The extraneous noises like the sound received outside the room and the sound produced by fans etc. are unnecessary noises. The acoustics of the buildings are also affected by resonance.

An auditorium is said to be good acoustics if the above unnecessary factors are reduced.

Requisites for Good Auditorium:

By using a large sounding board behind the speaker and facing the audience, necessary sound can be produced. Large polished wooden reflecting surface immediately above the speaker are also helpful. In such an arrangement, the listener will receive the reflected sound after 0.05 second. Hence there will be no confusion in the sound intensity.

The reverberation time of an auditorium should be brought to the optimum level. If there is no reverberation, then the auditorium is said to be a dead auditorium. This type of auditorium can be realized in practice by having open theaters. Music without reverberation loses its charm. So it is desirable to completely eliminate reverberation. But on the other hand, if the sound reverberates for a long time, the audience cannot differentiate each note. Hence it is necessary to reduce the time of reverberation. But it should not fall below a certain level.

The reverberation time for a hall can be adjusted by providing for absorption of sound in a number of ways. They are

1. Provision of windows and openings.
2. By covering the ceiling, walls and even backs of chairs with sound absorbing materials like fibre board.

3. Using a number of curtains
4. By carpeting the floor.
5. By decorating the walls with pictures and
6. Having a good - sized audience.

Focusing of sound gives unpleasant to the listener. This should be minimized by the following method.

1. There should be no curved surfaces
2. Ceiling should be low
3. Ornamental lamps, protruding parts etc are provided.
4. A paraboloidal reflecting surface is arranged at the focus of the speaker.

Echoes results from sound getting reflected at walls. Echoes are particularly trouble some in large halls. They can be removed almost entirely by making the surface, of the walls rough and by inclining the walls outward. However, a paint echoes are necessary for the enhancement of musical effects.

Resonance of a large hall is inversely proportional to the square root of the volume of the hall. Hence the volume should depend upon the frequency.

In a good hall no noise should reach from outside. This can be minimized by the following methods.

- 1 By avoiding opening for pipes and ventilators.
- 2 Using double doors and windows with separate frames.
- 3 Using heavy glass in doors, windows and ventilators.
- 4 By allotting proper places for doors and windows.
- 5 The machinery like type writers etc should be placed on absorbent pads.
- 6 Any engine inside the hall should be fitted on the floor with a layer of wood.
- 7 The floor should be covered with carpets.

Absorption coefficient

The absorption coefficient of a-material can be defined as the ratio of the sound energy aborted by the surface of the material to that of the total sound energy incident on the surface.

$$\text{Co-efficient of absorption} = \frac{\text{Sound energy absorbed by the surface}}{\text{Total sound energy incident on the surface}}$$

All sound waves falling on an open windows pass through. So an open windows can be assumed to be haves as a perfect absorber of sound. Hence the unit area of an open windows is taken as a standard unit of absorption.

Thus the absorption co-efficient of a material is defined as the ratio of the sound energy absorbed by certain area of the surface to that of an open window of the same area. The absorption coefficient is measured in open window unit written as O.W.U. or sapiens.

Absorption coefficient of a surface is also defined as the reciprocal of its which absorbs the same sound energy as absorbed by an unit area of an open window.

Measurement of absorption coefficient:

The absorption coefficient can be measured by the determination of reverberation time. When there is no absorbing, the reverberation time measured. Let it be T_1 , by applying Sabine's formula,

$$\frac{1}{T_1} = \frac{\Sigma aS_1}{0.16V} \quad - (1)$$

Now the absorbing material is put inside the room and the reverberation time is measured. Let it be T_2

$$\frac{1}{T_2} = \frac{\Sigma aS_1 + a_2S_2}{0.16V} \quad - (2)$$

$$\frac{1}{T_2} - \frac{1}{T_1} = \frac{a_2S_2}{0.16V}$$

$$a_2 = \frac{0.16V}{S_2} \times \frac{T_1T_2}{T_2 - T_1} \quad - (3)$$

Knowing S_2 and V , absorption coefficient a_2 can be calculated.

Reverberation

When a sound is produced in a building, it lasts too long after its production. It reaches the listener a number of times. The listener receives the direct sound from the source and also the reflected sound. Hence the intensity of sound decreases continuously. After some time, the intensity of sound decreases below the audible limit and then the sound disappears below the audible limit and then the sound disappears. This is called reverberation.

Reverberation Time

The reverberation time is defined as the time taken for the sound to fall below the minimum audibility measured from the instant when the source stopped sounding.

Sabine, using an organ pipe of frequency 512 Hz found that its sound became inaudible when its intensity fall to one millionth of its intensity just before stopping the organ pipe. The time taken for the intensity of sound wave to be reduced to one millionth of its initial value is called the reverberation time.

The reverberation time depends upon the size of the hall, loudness of the sound and upon the kind of music of sound for which time should have a optimum value. For a sound of frequency 512Hz, the best time of reverberation was found to be 1 to 1.5 seconds. For speech it is 0.5 second while for music it is between 2 and 2.5 seconds.

Sabine's formula

The relation between reverberation time, volume of the hall, area of the surface and its absorption coefficient is called Sabine's formula.

Let a_1, a_2, a_3 , etc be the absorption co-efficient at each reflection of the surface S_1, S_2, S_3 , etc in a room. Then the average value of the absorption co-efficient is given by \bar{a} is given by

$$\bar{a} = \frac{a_1 S_1 + a_2 S_2 + a_3 S_3 + \dots}{S_1 + S_2 + S_3}$$

$$\bar{a} = \frac{\sum a_i S_i}{S}$$

$$\text{Or } \sum a_i S_i = \bar{a} S$$

Where S is the total area of the surface.

By statistical method Jager two showed that the sound travels an average distance $4V/S$ between two successive reflections where V is the volume of the hall and S is the total area of the surface. If v is the velocity of sound wave, the time taken between two successive reflections is $4V/Sv$. Therefore the average number of reflection in time t is $Svt/4V$.

At a single reflection a is the fraction absorbed and $(1-\bar{a})$ is the fraction reflected. After two reflection, the fraction reflected is $(1-\bar{a})(1-\bar{a})$ i.e. $(1-\bar{a})^2$. If I_0 is the initial intensity of sound and I_t after t i.e. after $Svt/4V$ reflection, then

$$I_t = I_0 (1 - \bar{a})^{Svt/4V} \quad - (2)$$

According to the definition of reverberation time, when is equal to reverberation time T ,

$$I_t/I_0 = 10^{-6}$$

Then from equation (2)

$$10^{-6} = (1 - \bar{a})^{Svt/4V}$$

Taking logarithms on both sides

$$\log_e 10^{-6} = \frac{SvT}{4V} \log_e (1 - \bar{a})$$

$$\text{Or } T = \log_e 10^{-6} \times \frac{4V}{Sv \log_e (1 - \bar{a})}$$

$$= 2.3026 \log_{10} 10^{-6} \times \frac{4V}{Sv \log (1 - \bar{a})}$$

$$= \frac{2.3026 (-6) \times 4V}{Sv \times \log_e (1 - \bar{a})} \quad - (3)$$

Since \bar{a} is usually less than 1, we may expand $\log_e (1 - \bar{a})$ as a series.

$$\log_e (1 - \bar{a}) = a - \frac{\bar{a}^2}{2} - \frac{\bar{a}^3}{2}$$

Since a is much less than 1, the higher terms can be neglected.

$$\log_e (1 - \bar{a}) \approx -\bar{a} \quad - (4)$$

$$T = \frac{2.3026 (-6) \times 4V}{S(-\bar{a})}$$

If the velocity of sound $v = 340\text{m/s}$.

$$T = \frac{2.3026 (-6) \times 4V}{S(-\bar{a})} = \frac{0.16V}{\bar{S}a}$$

But $Sa = \sum a_i S_i$, the total absorbing power

$$T = \frac{0.165V}{\sum a_i S_i} \quad - (5)$$

Equation (5) gives the Sabine's reverberation formula the reverberation time is

1. Directly proportional to the volume of the hall.
2. Inversely proportional to the area of the walls.
3. Inversely proportional to the total absorption.

University Questions

2 Mark questions

1. What means ultrasonic waves?
2. What is piezo - electric effect?
3. Advantages and disadvantages of piezo - electric oscillator method.
4. Define absorption co-efficient?

5. Explain is reverberation?
6. What is reverberation time?
7. Define - Co-efficient of restitution.

5 Mark questions

1. Write properties of ultrasonic wave.
2. What are the essential features for good acoustics.
3. Which factors affects acoustics of building.
4. Explain measurement of absorption co-efficient.

10 Mark questions

1. Explain production of ultrasonic waves. Discuss piezoelectric oscillator.
2. Discuss applications of ultrasonic waves.
3. What are the requisites for good auditorium?
4. Derive Sabine's formula for reverberation.